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**377. Proposed by W. D. CAIRNS, Oberlin College.**

It is required to find a curve of the form  $y = x(x - a)(x - b)$  such that the abscissas of the maximum and minimum values, as well as  $a$  and  $b$ , shall be positive integers.

## MECHANICS.

When this issue was made up, no solutions had been received for numbers 289, 292-3, 295-299, 301, and 303.

**302. Proposed by CLIFFORD N. MILLS, Brookings, S. Dak.**

A ball is projected from a given point at a given inclination  $\beta$  towards a vertical wall; determine the velocity so that after striking the wall the ball may return to the point of projection.

## NUMBER THEORY.

When this issue was made up, no solutions had been received for numbers 215-16, 218, 220, and 223-226.

**226. Proposed by ELBERT H. CLARKE, Purdue University.**

If  $0!$  is taken equal to 1, and if  $k$  is any positive integer greater than or equal to 2, show that

$$\sum_{n=0}^{\infty} \frac{n!}{(k+n)!} = \frac{1}{(k-1)!} \cdot \frac{1}{(k-1)!}.$$

**227. Proposed by R. P. BAKER, University of Iowa.**

Show that every rational number can be expressed as a finite sum  $\sum_{n=m}^{n=m+k} \frac{a_n}{n}$ , where  $a_n$  is either 0 or 1 and  $m$  is any positive integer.

## SOLUTIONS OF PROBLEMS.

## ALGEBRA.

**409. Proposed by C. E. GITHENS, Wheeling, W. Va.**

Find integral values for the edges of a rectangular parallelopiped so that its diagonal shall be rational.

## II. SOLUTION BY ARTEMAS MARTIN, Washington, D. C.

On pp. 269-273 of the October, 1914, MONTHLY, W. C. Fells has solved a different problem from the one proposed. In making  $x^2 + y^2 = \square$ , he adds another condition not required.

Let  $x, y, z$  be the edges and  $d$  the diagonal of the parallelepiped; then we have to satisfy the equation

$$x^2 + y^2 + z^2 = d^2.$$

It is not necessary that  $x^2 + y^2$  be a square. Let us assume  $x = a, y = b, z + c = d$ , and we have

$$a^2 + b^2 + z^2 = (z + c)^2 = z^2 + 2cz + c^2,$$

which immediately gives

$$z = \frac{a^2 + b^2 - c^2}{2c} \quad \text{and} \quad d = \frac{a^2 + b^2 + c^2}{2c}.$$

Therefore

$$a^2 + b^2 + \left( \frac{a^2 + b^2 - c^2}{2c} \right)^2 = \left( \frac{a^2 + b^2 + c^2}{2c} \right)^2,$$

whatever be the values of  $a, b, c$ .

1. Taking  $a = 1, b = 2, c = 1$ , we find

$$1^2 + 2^2 + 2^2 = 3^2,$$

which is the *smallest* rational parallelepiped.

2. Taking  $a = 2, b = 3, c = 1$ , we have

$$2^2 + 3^2 + 6^2 = 7^2,$$

which is the smallest rational parallelepiped having its edges all different.

3. Taking  $a = 1, b = 4, c = 1$ , we get

$$1^2 + 4^2 + 8^2 = 9^2.$$

4. Let  $a = 2, b = 6, c = 2$ , and we have

$$2^2 + 6^2 + 9^2 = 11^2.$$

5. If  $a = 3, b = 4, c = 1$ , we get

$$3^2 + 4^2 + 12^2 = 13^2,$$

which, on p. 269, is stated to be "the smallest rational parallelepiped."

6. If we take  $a = 8, b = 9, c = 5$ , we will get

$$8^2 + 9^2 + 12^2 = 17^2.$$

7. Taking  $a = 4, b = 8, c = 2$ , we have

$$4^2 + 8^2 + 19^2 = 21^2.$$

8. If  $a = 3, b = 4, c = 3$ , then we get

$$12^2 + 15^2 + 16^2 = 25^2.$$

And so on, there being an infinite number of parallelepipeds whose edges and solid diagonals are rational integers.

The condition  $x^2 + y^2 + z^2 = \square$  can be satisfied in many ways.—See *Mathematical Magazine*, Vol. II., No. 5 (October, 1891), pp. 71–74. Other methods are given in a forthcoming paper which will appear in the third part of No. 12, Vol. II., of the *Mathematical Magazine*.

**416A. Proposed by H. O. HANSON, East Elmhurst, N. Y.**

Find the  $n$ th term and the sum of  $n$  terms of the series obeying the relation  $u_i = u_{i-1} + 2u_{i-2}$  in terms of  $n$  and the first two terms,  $u_1$  and  $u_2$ , these two terms being arbitrary.

This problem was incorrectly numbered 416.